

Robust Sensorless Speed Control of SPIM based on MRAS Sliding Mode Adaptation Mechanism

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Abstract. *With the aim to achieve high dynamic performances under system uncertainties and load torque even at very low speed range, this paper presents a novel robust sensorless speed vector control strategy for single phase induction motor (SPIM). The proposed strategy includes a speed controller and a model reference adaptive system (MRAS) estimation algorithm of rotor speed both are based sliding mode technique. The proposed speed controller, qualified as proportional integral sliding mode controller (PISMC), use an online self adaptive switching gain. The speed PISMC is designed without need of uncertainties or external disturbances bounds' determination usually mandatory for the design of sliding mode controllers. As well, the developed MRAS adaptation mechanism is formed of two control law terms, equivalent and nonlinear one with fixed switching gain. Therefore, sublimite algorithms to estimate parameter variation at very low speed range in order to maintain high drive performance are not required. The stability of both of them is proved using Lyapunov approach. Extensive computer simulations highlight the effectiveness of the proposed sensorless speed control strategy under parameters variations and load torque.*

Keywords. *Chattering, model reference adaptive system, sensorless speed control, signal phase induction motor, sliding mode control,*

1. Introduction

The variable-speed drives have been integrated recently in domestic and industrial applications using single phase induction motors (SPIMs). In order to ameliorate the SPIM performances, many conventional controller structures were proposed in literature [1]-[4]. Recently, the sliding mode technique has been investigated to drive the SPIM speed in [5]-[7]. It is well known that variable sensor-less speed control, give us installation cost reduction, energy saving and avoid decreased system reliability [1]-[3]. Therefore, they still up to now an interest research area. Due to their design simplicity, the speed estimator techniques based on model reference adaptive system (MRAS) are the mainly used approaches. Generally, they provide satisfactory speed estimation in the high and medium speed regions. However at low speed range, the MRAS techniques present significant deterioration on speed performance due to parameters variation, pure integration and measurement noise. In order to maintain good dynamic performance, simultaneous accurate knowledge or estimation of speed and at least one of motor parameters (rotor time constant, stator resistance ...) is required [8],[9]. Therefore, classical MRAS algorithms become more complex and increase the installation cost which represents a severe constraint for real time implantation.

The paper proposes a new robust sensorless speed vector control strategy dedicate to drive a SPIM. Since there are some significant advantages associated with sliding mode control technique [10], [11], this strategy incorporates a robust speed controller and a new MRAS rotor speed algorithm estimation based sliding mode. The proposed MRAS algorithm cooperates with the speed PISMC, to exhibit high performance sensorless speed control without adding sublimite blocs. This purpose is achieved by the design of a MRAS adaptation mechanism made up of an equivalent control law associated with a fixed switching gain law. To take care of parameter uncertainties and load torque variation, the developed speed PISMC call for an online self adaptive switching gain. As well, the chattering effect can be clearly reduced [12]-[14]. So, the two designed elements operate in manner to obtain a very high sensorless control performance at parameters variation, load condition reversal speed and very low speed range.

The paper is organized as follows; section 3 briefly reviews the indirect rotor field control strategy used to drive the SPIM. Sections 4 and 5 deal with the proposed speed PISMC and MRAS algorithm design. Simulation results are presented and discussed in section 6.

2. Indirect field oriented control strategy of SPIM.

The main idea behind the field oriented strategy is to align the flux vector along a chosen direction so that the behaviours of the SPIM look as a DC motor behaviours', where the flux and the torque can be controlled independently by the direct and quadratic currents. Its must be shown that a symmetric model of the SPIM have to be established before applying the field oriented strategy.

The block diagram of the proposed indirect rotor field oriented control strategy is presented in Figure 1. It involves an adaptive sliding mode speed controller and a MRAS speed estimator using a sliding mode adaptive mechanism. The output of the speed controller gives the desired quadratic current (or electromagnetic torque) reference. In addition, the strategy requires the design of two others controllers to drive the main and auxiliary winding currents (which do not take part of this study), where their outputs are the stator voltage vector.

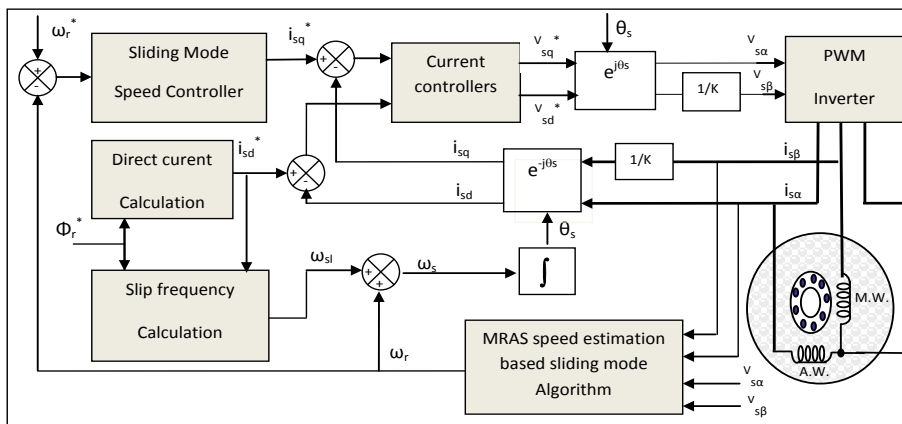


Fig. 1: Block diagram of the proposed rotor field oriented control strategy

3. Robust speed control with self adaptive sliding gain design

The SPIM mechanical equation is:

$$J\dot{\omega} + f\omega = n_p (T_e - T_L) \quad (1)$$

Applying the vector control principle, the desired electromagnetic torque expression becomes:

$$T_e = n_p \frac{M_{sr}}{JL_r} \varphi_r^* i_{qs} \quad (2)$$

Then, the mechanical equation of the SPIM can be arranged as follows:

$$\dot{\omega} = -a\omega + b i_{qs} - c \quad (3)$$

where: $a = \frac{f}{J}$; $b = n_p^2 \frac{M_{sr}}{JL_r} \varphi_r^*$ and $c = \frac{n_p}{J} T_L$

Considering the previous mechanical equation with parameters uncertainties as in [9]:

$$\dot{\omega} = -(a + \Delta a)\omega + (b + \Delta b) i_{qs} - (c + \Delta c) \quad (4)$$

Where Δa , Δb and Δc : represent the uncertainties terms. Defining the tracking rotor speed error as follows:

$$e = \omega^* - \omega \quad (5)$$

The first derivative form of the tracking rotor speed error with respect to time is:

$$\dot{e} = \dot{\omega}^* - \dot{\omega} = -ae + u + d \quad (6)$$

where: $u = \dot{\omega}^* + a\omega^* - b i_{qs}$ and: $d = \Delta a\omega - \Delta b i_{qs} + (c + \Delta c)$

This arrangement is due to the fact that the quadratic current represents an implicit control signal in the field oriented control strategy. The term d made up of the combination system uncertainties and external disturbances (load torque), is considered always bounded. As well, we considered that there is an unknown positive gain called adaptive switching gain and noted G verifying:

$$G > d_{\max} + \varepsilon \quad \text{where: } d_{\max} > |d| \quad \text{and} \quad \varepsilon > 0 \quad (7)$$

The switching gains chosen in classical sliding mode control schema; depend usually on the upper limit of the term d . where the determination is not always possible or at least simple. Therefore and with the aim to elaborate a robust control law to drive the speed SPIM even with the presence of the above disturbance, we propose an adaptive speed PISM. As will be shown next, the design of this controller doesn't require the knowledge of uncertainties and disturbances upper limit. The synthesis of the proposed speed PISM is based on the following proportional integral sliding surface, where k_w is a positive constant gain:

$$S_\omega = e + k_w \int_0^t e d\tau \quad (8)$$

Therefore, we propose the following control law U composed by a linear term and a discontinue one:

$$U = -(k_\omega - a)e - (1 + \eta)\hat{G} \cdot \text{sgn}(S_\omega) \quad (9)$$

The adaptive switching gain G is calculated according to the following adapted law:

$$\hat{G} = \int_0^t (1 + \eta)|S_\omega| d\tau + G_0 \quad \text{where: } G_0 > 0 \quad \text{and} \quad \eta > 0 \quad (10)$$

The compensation of disturbances and parameter variation effects is assured by the term η . Where, the term G_0 represents the sliding gain initial value and is fixed respecting the desired speed convergence.

Expression (10) proves that the proposed PISMC does not require the knowledge of uncertainties and disturbances bounders, to determine the switching gain value as the classical sliding mode technique. The variation of switching function value will be regarded as an eventual external disturbances or parameter uncertainties. Then, the switching gain will be online updated according to the instantaneous sliding surface value. Moreover and when the state spaces are far from of the desired sliding surface, the switching gain value will increase to force them to reach the desired sliding surface more rapidly.

Stability Proof:

Let us consider the subsequent candidate Lyapunov function V_ω :

$$V_\omega = \frac{1}{2} (S_\omega^2 + \tilde{G}^2) \text{ where: } \tilde{G} = \hat{G} - G \quad (11)$$

Differencing (11) with respect to time gives:

$$\dot{V}_\omega = S_\omega \dot{S}_\omega + \tilde{G} \dot{\tilde{G}} = S_\omega (\dot{e} + k_\omega e) + (\hat{G} - G) \dot{\hat{G}} \quad (12)$$

Replacing the derivative terms by their expressions:

$$\dot{V}_\omega = S_\omega (d - \eta \hat{G} \text{sgn}(S_\omega)) + \eta (\hat{G} - G) |S_\omega| \quad (13)$$

Taking account the expressions (7) we get:

$$\dot{V}_\omega \leq -\eta (\varepsilon + d_{\max}) |S_\omega| \quad (14)$$

Then:

$$\dot{V}_\omega \leq 0 \quad (15)$$

Therefore, and under condition that uncertainties and load torque are bounded, the proposed control law make the proportional integral sliding surface S_w attractive.

In addition, equation (8) provides that, at the sliding surface, speed tracking error tends exponentially to zero. It must be signaled that the sign function is substituted by smother continuous ones in order to reduce the chattering effect [14], [15].

4. MRAS Rotor speed estimator based sliding mode design

The basic concept of MRAS estimator technique is the use of two models (reference model and adjustable model), and an adaptation mechanism to estimate the system states or parameters. The two following expressions of the rotor flux derived from the stator voltage SPIM equations expressed in the stationary reference frame, which does not include the rotor speed term are consequently considered as a reference model.

$$\dot{\phi}_{r\alpha} = \frac{L_r}{M_{sd}} [V_{s\alpha} - (R_{sd} + pL_{sd} \sigma_{sd}) i_{s\alpha}] \quad (16)$$

$$\dot{\phi}_{r\beta} = \frac{L_r}{M_{sq}} [V_{s\beta} - (R_{sq} + pL_{sq} \sigma_{sq}) i_{s\beta}] \quad (17)$$

However, the two other expressions of the rotor flux resulting from the rotor voltage equations of the SPIM expressed in the stationary reference frame involve the rotor fluxes and the speed term. These equations serve as the adjustable model:

$$\dot{\hat{\varphi}}_{ra} = -\frac{1}{T_r} \hat{\varphi}_{ra} - \hat{\omega} \hat{\varphi}_{rb} + \frac{M_{sr\alpha}}{T_r} i_{s\alpha} \quad (18)$$

$$\dot{\hat{\varphi}}_{rb} = \hat{\omega} \hat{\varphi}_{ra} - \frac{1}{T_r} \hat{\varphi}_{rb} + \frac{M_{sr\beta}}{T_r} i_{s\beta} \quad (19)$$

As shown, both reference model and adjustable model equations depend on some motor parameters, as a result an ordinary parameters variation can affect the MRAS estimator performances. To overcome this problem, many MRAS techniques necessitate the simultaneous precise estimation of speed and parameter variation.

The improvement cost sometimes represents a severe constraint for real time implantation. Thus, we propose a novel sliding mode rotor speed MRAS estimator ensuring good behaviors even at very low speed range. The developed sliding mode MRAS schema is free of online parameter variation estimation, therefore a significant improvement in time implementation cost will be registered.

Fig. 2 gives the structure of the proposed MRAS speed estimation. Where, the tuning signal is the input of the designed adaptation mechanism based sliding mode. This delivers the value of the estimated rotor speed to actualize the adjustable model.

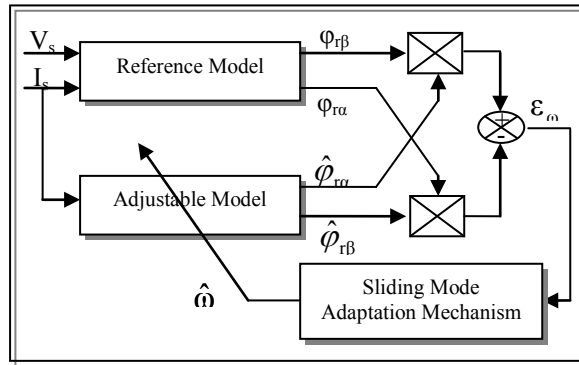


Fig. 2: Schema of the proposed MRAS based sliding mode

The adopted speed tuning signal ϵ_ω is expressed by the following equation:

$$\epsilon_\omega = \hat{\varphi}_{ra} \varphi_{r\beta} - \varphi_{ra} \hat{\varphi}_{r\beta} \quad (20)$$

Using the previous speed tuning signal (20), and defining a proportional sliding surface S as:

$$S = k\epsilon_\omega \quad \text{with } k > 0 \quad (21)$$

When the desired state spaces reach the switching surface and stay there, this gives:

$$S = \dot{S} = 0 \quad (22)$$

Therefore, the error dynamic can be described by the following equation, meaning that it will be forced to exponentially tend to zero.

$$\dot{\epsilon}_\omega = -k\epsilon_\omega \quad (23)$$

Differentiating equation (20) yields:

$$\dot{\epsilon}_\omega = \dot{\varphi}_{r\beta} \hat{\varphi}_{ra} + \varphi_{r\beta} \dot{\hat{\varphi}}_{ra} - \dot{\varphi}_{ra} \hat{\varphi}_{r\beta} - \varphi_{ra} \dot{\hat{\varphi}}_{r\beta} \quad (24)$$

Replacing the derivative terms by their expressions, this equation can be written as:

$$\dot{\epsilon}_\omega = k_1 - k_2 \hat{\omega}_r \quad (25)$$

With:

$$k_1 = \hat{\varphi}_{r\beta} \hat{\varphi}_{r\alpha} - \hat{\varphi}_{r\alpha} \hat{\varphi}_{r\beta} + \frac{1}{T_r} (M_{srd} i_{sa} \varphi_{r\beta} - M_{srq} i_{s\beta} \varphi_{r\alpha}) + \frac{1}{T_r} (\hat{\varphi}_{r\beta} \varphi_{r\alpha} - \hat{\varphi}_{r\alpha} \varphi_{r\beta})$$

$$\text{and: } k_2 = \varphi_{r\alpha} \hat{\varphi}_{r\alpha} + \varphi_{r\beta} \hat{\varphi}_{r\beta}$$

The rotor speed can be estimated using the following proposed adaptation control law made up equivalent term and non linear term:

$$\hat{\omega}_r = U_{eq} + U_d \quad (26)$$

$$\text{where: } U_{eq} = \frac{k_1 + \epsilon_\omega}{k_2} \quad \text{and } U_d = \frac{G_1}{k_2} \text{sign}(S) \quad G_1 > 0$$

The estimator speed convergence depends on the fixed gain G_1 choice. As well as the previous section and in order to reduce the chattering effect on the estimated speed value, the sign function is changed by a smoother continuous one.

Stability Proof:

The stability of the proposed sliding mode control law describing the adaptation mechanism can be elaborated using Lyapunov theory. So, defining the candidate Lyapunov function V as:

$$V = \frac{1}{2} S^2 \quad (27)$$

The expression of the first time derivative of the candidate Lyapunov function is:

$$\dot{V} = S(k\dot{\epsilon}_\omega) \quad (28)$$

Taking into account expressions (25) and (26), we get:

$$\dot{V} = -k^2 \epsilon_\omega^2 - kG_1 |S| \leq 0 \quad (29)$$

Therefore the proposed control law is stable and the switching function S is always attractive. It's clear that the speed observability depend on the term k_2 (due to the presence of k_2 in the denominator control expression). To avoid the divergence of the designed MRAS algorithm, we can permit magnetizing of the SPIM before starting up the speed estimation or by adding a small positive value to k_2 .

5. Simulations results

Simulation studies have been performed in order to validate the proposed SPIM drive strategy. The used SPIM, speed PISMC and the sliding mode MRAS algorithm parameters are listed in the following Tables. To illustrate the effectiveness of the proposed speed PISMC and the convergence of the estimated MRAS speed to the measured value, several tests were investigated. The first test is devoted to verify the

SPIM behaviours at nominal speed, so, the SPIM started without load torque, a nominal load torque is applied at 1s and a reversal speed at 1.5 s is considered.

Table 1.
SPIM parameters

R_{sd} : 2.473 Ω	Rated power : 1.1 kW
R_{sq} : 6.274 Ω	Rated voltage : 220 V
R_r : 5.514 Ω	Rated current : 5.1 A
L_{sd} : 0.0904 H	Rated frequency : 50 Hz
L_{sq} : 0.1099 H	Number of pole pairs: 2
L_r : 0.0904 H	Rated speed : 1430 rpm
M_{srd} : 0.0817	J : $0.9 \cdot 10^{-3}$ kg.m ²
M_{srq} : 0.0715	f : $1.2 \cdot 10^{-3}$ N.m.s.rad ⁻¹

Table 2.
PISMC and MRAS parameters

PISMC parameters	MRAS parameters
k_{ω} : 0.001	k : 0.01
$G(0)$: 15	G_1 : 5
η : 100	

The second one is reserved to evaluate the performances of the proposed strategy at very low speed range (10 rad/s). Where, the SPIM started with nominal load torque, and a reversal speed is considered at 1.5s.

The robustness of the proposed strategy against parameter variations is studied in the two latest tests; in witch a variation of $\pm 25\%$ of the total inertia J is considered.

Fig. 3 display that operating at nominal speed, the proposed speed PISMC and speed MRAS estimator exhibit good tracking performances and fast response without steady error or overshoot. As well, the estimated speed error is cancelled after 0.5 s. The applied nominal load at 1s has no significant effect in the speed response or estimation.

As shown in Fig. 6, the MRAS algorithm still exhibit a good tracking error, even at very low speed range. Where, the speed PISMC, presents a neglected steady state error. This is due to the approximation of sign function with smother continuous one used in order to reduce the chattering effects.

The evolution of the self adaptive switching gain of the designed speed PISMC is given by Fig. 4b, 7b, 10b and 13b. As illustrated in different cases, the value of adaptive switching gains changes accordingly to the switching function variation, to compensate the disturbance influence (load torque, reversal speed, parameter variation...). As well, the proposed MRAS adaptation mechanism law succeed to carry the signal tuning to zero (Fig. 5a, 8a, 11a and 14a).

In all tests, the designed control law imposes the reaching of the sliding surface in finite time and then the speed stay there even in the presence of external disturbances or total inertia variation (Fig. 4a, 7a, 10a and 13a).

Finally, Fig. 5b and Fig. 8b show that the generated electromagnetic torque, typically retraces the chosen load torque scenario in both tests. This is done without need of load torque measurement or estimation. A steady error is notated in the generated

electromagnetic torque reference in the tow latest tests (Fig. 11b and Fig. 14b). However, this has no effect on the speed SPIM response.

Test 1: Nominal speed

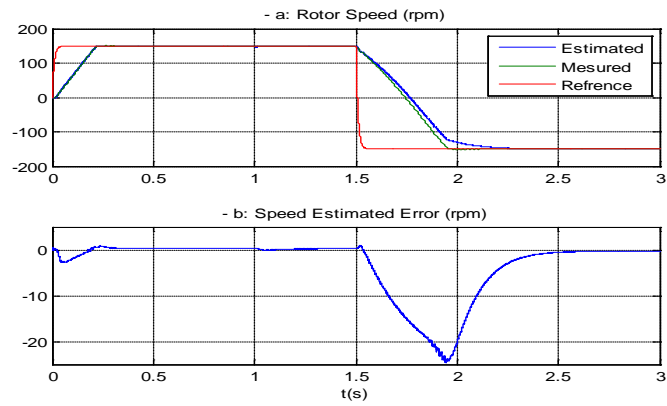


Fig. 3: - a: reference, measured and estimated speed / - b: estimated error rotor speed

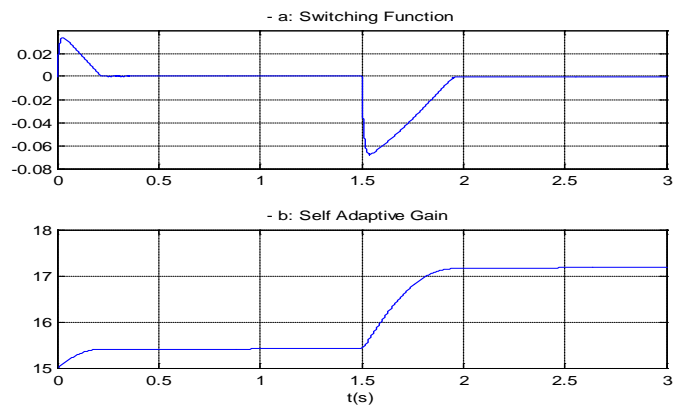


Fig. 4: - a: Switching function / - b: Self adaptive switching gain

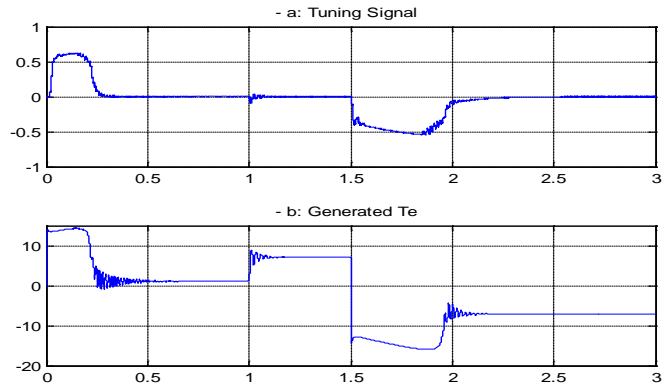


Fig. 5: -a: MRAS tuning signal / -b: Generated electromagnetic torque reference
Test 2: Very low speed

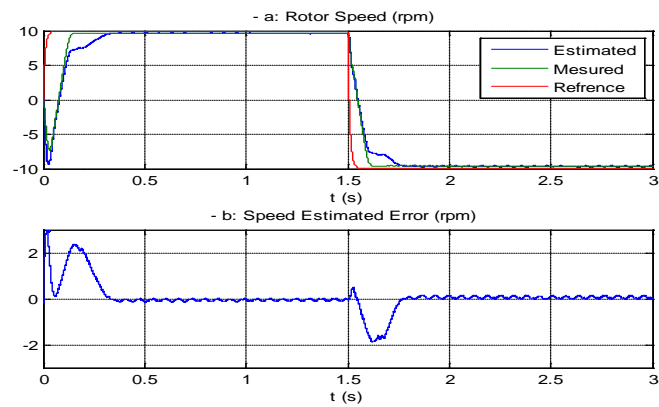


Fig. 6: - a: reference, measured and estimated speed / -b: estimated error rotor speed

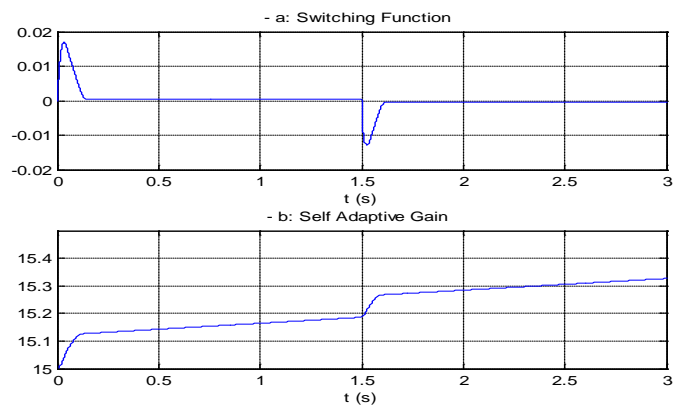


Fig. 7: - a: Switching function / - b: Self adaptive switching gain

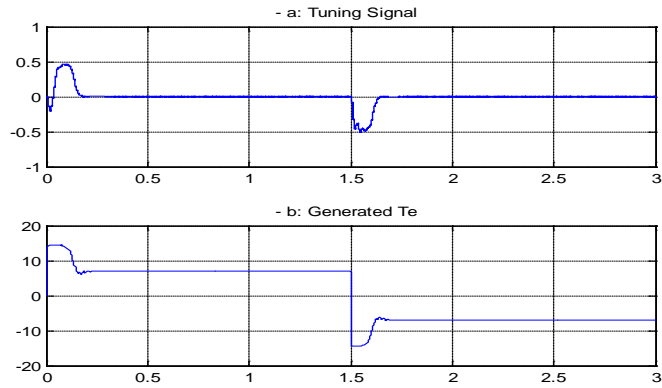


Fig. 8: -a: MRAS tuning signal / -b: Generated electromagnetic torque reference
Test 3: SPIM behaviours (+ 25% of total inertia)

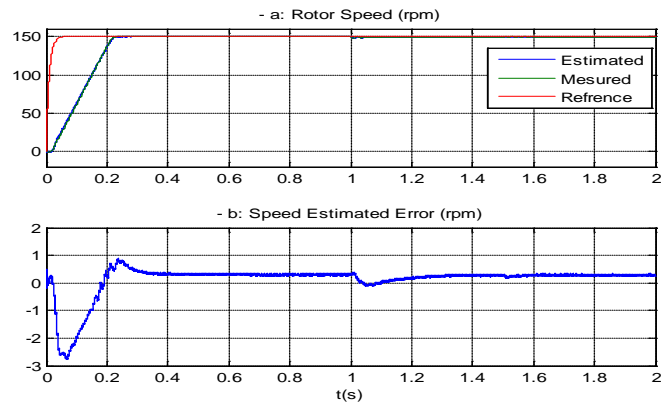


Fig. 9: - a: reference, measured and estimated speed / -b: estimated error rotor speed

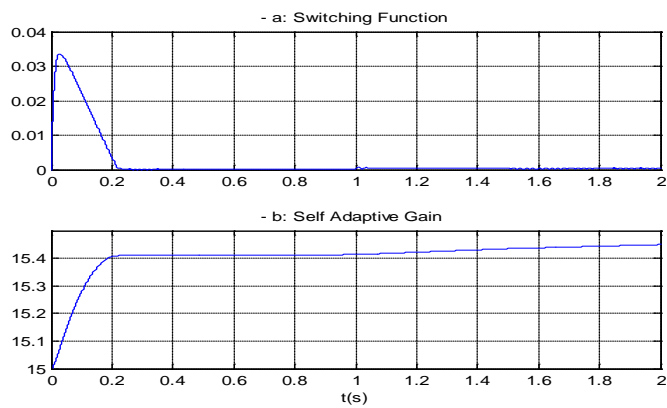


Fig. 10: - a: Switching function / - b: Self adaptive switching gain

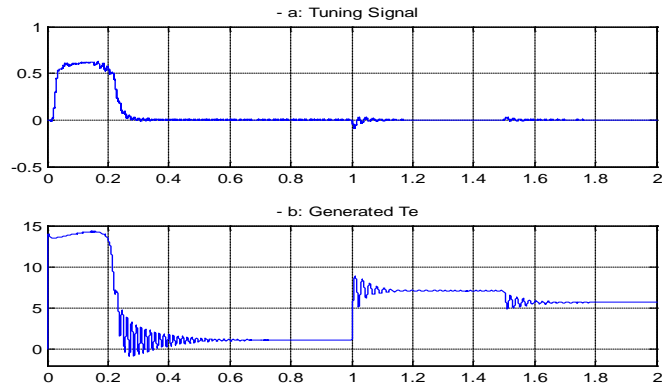


Fig. 11: -a: MRAS tuning signal / -b: Generated electromagnetic torque reference
Test 4: SPIM behaviours (- 25% of total inertia)

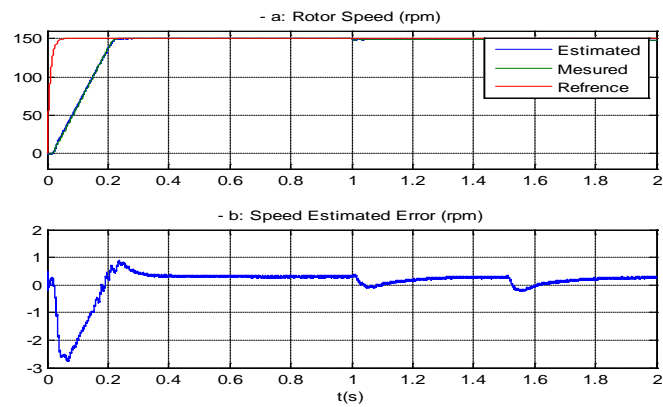


Fig. 12: - a: reference, measured and estimated speed / -b: estimated error rotor speed

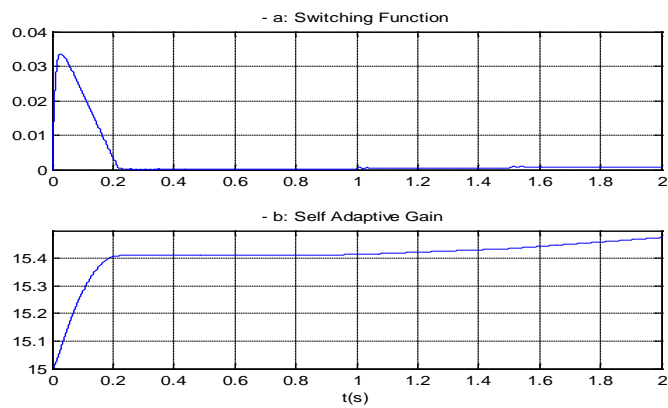


Fig. 13: - a: Switching function / - b: Self adaptive switching gain

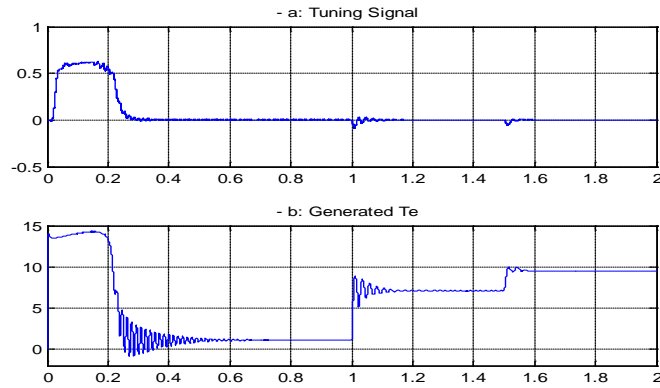


Fig. 14: -a: MRAS tuning signal / -b: Generated electromagnetic torque reference

6. Conclusion

In this paper, a sensorless rotor field oriented control strategy is proposed to drive a SPIM operating even at very low speed range. This strategy integrates a robust speed PISM and a MRAS sliding mode adaptation mechanism. Based on very simple online self adaptive switching gain, the developed PISM control law did not require the knowledge of uncertainties bounds as classical sliding mode controllers to guarantee the desired performances. As well, the main advantage of the proposed MRAS speed estimator is that doesn't necessitate a sublimite algorithm for parameter estimation to have accurate speed estimation at very low speed range.

Both of the speed PISM and the MRAS algorithm stability was derived by Lyapunov approach. Finally, and by means of simulation results, it has been showed that the designed robust sensorless speed SPIM drive, exhibit very good tracking trajectory and high performance speed estimation under load conditions and parameter variations even at very low speed range.

Nomenclature

- $v_{s\alpha}, v_{s\beta}$: stator voltages in stationary reference frame;
- $i_{s\alpha}, i_{s\beta}$: stator currents in stationary reference frame;
- $\phi_{r\alpha}, \phi_{r\beta}$: rotor fluxes in stationary reference frame;
- $L_{sd}, L_{sq}, L_r, M_{srd}, M_{srq}$: stator and rotor self and mutual inductances;
- $R_{sd}, R_{sq},$ and R_r : stator and rotor resistances;
- ω, ω_{sl} : rotor and slip angular frequency;
- T_e, T_L : electromagnetic and load torque;
- f : friction coefficient;
- J : total inertia;

- n_p : pole pairs number.

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